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ABSTRACT

The Colorado Model Content Standards for Mathematics were designed to serve as a guide for school districts as they define standards that will enable every Colorado student to develop the mathematical literacy needed for citizenship and employment in the 21st century. A new vision is provided of the content students should study in order to achieve the six goals of becoming mathematical problem solvers, learning to communicate mathematically, learning to reason mathematically, making mathematical connections, becoming confident of their mathematical abilities, and learning the value of mathematics. Six standards are presented with their rationale and specific expectations for students completing grades K-4, 5-8, and 9-12. (ASK)

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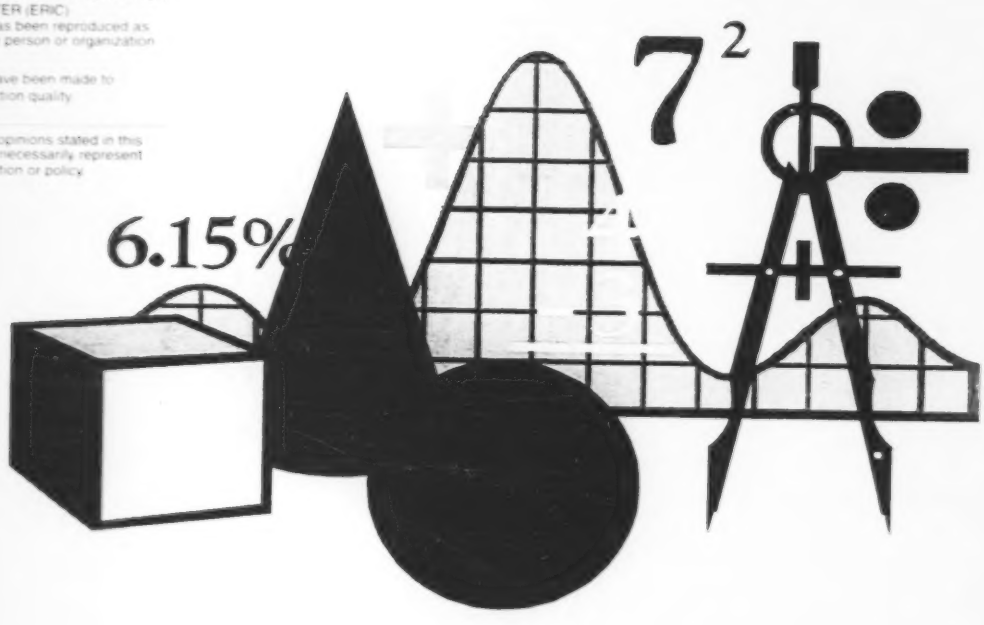

MATHEMATICS

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INTRODUCTION

Colorado Model Content Standards for Mathematics

Responsible and productive members of today's technological society need to have a broad, connected, and useful knowledge of mathematics. The Colorado Model Content Standards for Mathematics are designed to serve as a guide for school districts as they define standards that will enable every Colorado student to develop the mathematical literacy needed for citizenship and employment in the 21st century.

> "Today's students will live and work in the 21st century, in an era dominated by computers, by worldwide communication, and by a global economy. Jobs that contribute to this economy will require workers who are prepared to absorb new ideas, to perceive patterns, and to solve unconventional problems. Mathematics is the key to opportunity for these jobs."[1]

Mathematics is not simply a collection of facts and procedures, and doing mathematics is not simply recalling these facts, nor performing memorized procedures. Mathematics is a coherent and useful discipline that has expanded dramatically in the last 25 years. The mathematics students study in school must reflect these changes, and the ways students study mathematics must capitalize on the growth in our understanding of how students learn.

> "There has been a mentality that you have to be ... special to be successful in mathematics, that you have to be the best and the brightest. Well, we are demystifying mathematics. We can no longer say that there is any segment of society that doesn't need mathematics."[2]

Three questions have guided the development of the Colorado Model Content Standards for Mathematics: What is mathematics? What does it to mean to know, use, and do mathematics? What mathematics should *every* Colorado student learn?

Responses to these questions have resulted in six goals, adapted from those of the National Council of Teachers of Mathematics[3], that serve as the framework for the Colorado Model Content Standards for Mathematics. The six goals that Colorado students should reach are stated on the following page.

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[1] L. Steen, (1989), "Teaching Mathematics for Tomorrow's World", *Educational Leadership*, 47: 18-22.

[2] Quote by Iris Carl found in A. Wheelock, (1992), *Crossing the Tracks*, (New York: The New Press).

[3] National Council of Teachers of Mathematics, (1989), *Curriculum and Evaluation Standards for School Mathematics*, (Reston, VA: author).

Six Goals for Colorado Students of Mathematics

- ***Become mathematical problem solvers.*** To be problem solvers, students need to know how to find ways to reach a goal when no routine path is apparent. To develop the flexibility, perseverance, and wealth of strategies that are characteristic of good problem solvers, students need to be challenged frequently and regularly with non-routine problems, including those they pose themselves.

- ***Learn to communicate mathematically.*** The development of students' power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations where students have an opportunity to read, write, and discuss ideas in the language of mathematics. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking.

- ***Learn to reason mathematically.*** Students who reason mathematically gather data, make conjectures*, assemble evidence, and build an argument to support or refute these conjectures. Such processes are fundamental to doing mathematics.

- ***Make mathematical connections.*** The study of mathematics should provide students with many opportunities to make connections among mathematical ideas (for example, the connection between geometric and algebraic concepts) and among mathematics and other disciplines (for example, art, music, psychology, science, business). The curriculum should portray mathematics as an integrated whole that permeates activities both in and out of school. These connections make mathematics meaningful and useful to each Colorado student.

- ***Become confident of their mathematical abilities.*** As a result of studying mathematics, students need to view themselves as capable of using their growing mathematical power to make sense of new problem situations in the world around them. School mathematics must endow all students with a realization that doing mathematics is a common human activity. Students learn to trust their own mathematical thinking by having numerous and varied experiences.

- ***Learn the value of mathematics.*** In addition to providing the tools to solve problems, mathematics provides a way of thinking about and understanding the world around us. Students should have numerous and varied opportunities to think mathematically about their world. They should also explore the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society.

The following Colorado Model Content Standards for Mathematics provide a new vision of the content students should study in order to achieve these goals. The standards reinforce the need for technical skills, long a goal of school mathematics, and also the need to know when to apply them and why they work. They also broaden considerably the context in which these technical skills might be attained. Students who have a working knowledge of the mathematics in each of these standards will be better able to reason critically, vote responsibly, and work productively in today's complex world.

*A glossary of terms can be found on page Mathematics - 22.

Colorado Model Content Standards for Mathematics

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Colorado Model Content Standards

MATHEMATICS

1. **Students develop number sense* and use numbers and number relationships in problem-solving situations* and communicate the reasoning used in solving these problems.**

2. **Students use algebraic methods* to explore, model*, and describe patterns* and functions* involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.**

3. **Students use data collection and analysis, statistics*, and probability* in problem-solving situations and communicate the reasoning used in solving these problems.**

4. **Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.**

5. **Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.**

6. **Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic*, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.**

STANDARD 1:

Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.

In order to meet this standard, a student will

- construct and interpret number meanings through real-world experiences* and the use of hands-on materials;
- represent and use numbers in a variety of equivalent forms (*for example, fractions, decimals, percents, exponents*, scientific notation**);
- know the structure and properties of the real number system* (*for example, primes*, factors, multiples, relationships among sets of numbers*); and
- use number sense, including estimation and mental arithmetic, to determine the reasonableness of solutions.

RATIONALE

Numbers play a vital role in our daily lives, from cooking to reading the newspaper to performing jobs. Because we use numbers to measure, to count, to order, and to label, it is important to understand the many uses of numbers. These include knowing both the symbols for and the meanings of various kinds of numbers, including whole numbers, fractions, decimals, percents, roots, exponents, logarithms, and scientific notation.*

Number sense is "common sense" about numbers. Students with number sense recognize the relative magnitudes of numbers and relationships between numbers; for example, 1/2 is equivalent to .5 and 50%. In addition, they have references for measures of common objects and situations in the environment. They know how much a million is and how much a loaf of bread costs. Developing number sense strengthens students' ability to acquire basic facts, to solve problems, and to determine the reasonableness of results.*

GRADES K-4

In grades K-4, what students know and are able to do includes

- demonstrating meanings for whole numbers, and commonly-used fractions and decimals *(for example,* $1/3, 3/4, 0.5, 0.75$*),* and representing equivalent forms of the same number through the use of physical models, drawings, calculators, and computers;
- reading and writing whole numbers and knowing place-value concepts and numeration through their relationships to counting, ordering, and grouping;
- using numbers to count, to measure, to label, and to indicate location;
- developing, testing, and explaining conjectures* about properties of whole numbers, and commonly-used fractions and decimals *(for example,* $1/3, 3/4, 0.5, 0.75$*);* and
- using number sense to estimate and justify the reasonableness of solutions to problems involving whole numbers, and commonly-used fractions and decimals *(for example,* $1/3, 3/4, 0.5, 0.75$*).*

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

- demonstrating meanings for integers*, rational numbers*, percents, exponents, square roots*, and pi (π) using physical materials and technology in problem-solving situations;
- reading, writing, and ordering integers, rational numbers, and common irrational numbers* such as $\sqrt{2}$, $\sqrt{5}$, and π;
- applying number theory concepts *(for example, primes, factors, multiples)* to represent numbers in various ways;
- using the relationships among fractions, decimals, and percents, including the concepts of ratio and proportion, in problem-solving situations;
- developing, testing, and explaining conjectures about properties of integers and rational numbers; and
- using number sense to estimate and justify the reasonableness of solutions to problems involving integers, rational numbers, and common irrational numbers such as $\sqrt{2}$, $\sqrt{5}$, and π.

GRADES 9-12

As students in grades 9-12 extend their knowledge, what they know and are able to do includes

- demonstrating meanings for real numbers, absolute value*, and scientific notation using physical materials and technology in problem-solving situations;
- developing, testing, and explaining conjectures about properties of number systems and sets of numbers; and
- using number sense to estimate and justify the reasonableness of solutions to problems involving real numbers.

For students continuing their mathematics education beyond these standards, what they will know and are able to do **may** include

- investigating limiting processes by examining infinite sequences and series; and
- explaining relationships among real numbers, complex numbers*, and vectors* using models.

STANDARD 2:

Students use algebraic methods to explore, model, and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.

In order to meet this standard, a student will

- identify, describe, analyze, extend, and create a wide variety of patterns in numbers, shapes, and data;
- describe patterns using mathematical language;
- solve problems and model real-world situations using patterns and functions;
- compare and contrast different types of functions; and
- describe the connections among representations of patterns and functions, including words, tables, graphs, and symbols.

RATIONALE

The study of patterns, functions, and algebra helps learners to recognize and generalize patterns; identify and clarify functional relationships; and represent and manipulate these relationships verbally, numerically, symbolically, and graphically. Symbolic representation, including the many interpretations of the concept of variable*, is important but only one of many ways to represent patterns and functions. Students who are adept at identifying and classifying patterns and functional relationships are better able to use these relationships in real situations, both in school and out. The portrayal of functions and algebra in this standard is broader, deeper, more connected, and more useful to learners than in the traditional high school algebra curriculum.*

Because the understandings developed through this standard are critical to success in mathematics and to the appropriate use of quantitative reasonings in other disciplines, students should explore and use the ideas of functions, patterns, and algebra from kindergarten through 12th grade.

GRADES K-4

In grades K-4, what students know and are able to do includes

- reproducing, extending, creating, and describing patterns and sequences using a variety of materials *(for example, beans, toothpicks, pattern blocks, calculators, unifix cubes, colored tiles);*
- describing patterns and other relationships using tables, graphs, and open sentences*;
- recognizing when a pattern exists and using that information to solve a problem; and
- observing and explaining how a change in one quantity can produce a change in another *(for example, the relationship between the number of bicycles and the numbers of wheels).*

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

- representing, describing, and analyzing patterns and relationships using tables, graphs, verbal rules, and standard algebraic notation;
- describing patterns using variables, expressions, equations, and inequalities in problem-solving situations;
- analyzing functional relationships to explain how a change in one quantity results in a change in another *(for example, how the area of a circle changes as the radius increases, or how a person's height changes over time);*
- distinguishing between linear* and nonlinear functions through informal investigations; and
- solving simple linear equations in problem-solving situations using a variety of methods *(informal, formal, graphical)* and a variety of tools *(physical materials, calculators, computers).*

GRADES 9-12

As students in grades 9-12 extend their knowledge, what they know and are able to do includes

- modeling real-world phenomena *(for example, distance-versus-time relationships, compound interest, amortization tables, mortality rates)* using functions, equations, inequalities, and matrices*;
- representing functional relationships using written explanations, tables, equations, and graphs, and describing the connections among these representations;
- solving problems involving functional relationships using graphing calculators and/or computers as well as appropriate paper-and-pencil techniques;
- analyzing and explaining the behaviors, transformations*, and general properties of types of equations and functions *(for example, linear, quadratic*, exponential*);* and
- interpreting algebraic equations and inequalities geometrically and describing geometric relationships algebraically.

For students continuing their mathematics education beyond these standards, what they know and are able to do **may** include

- using rational, polynomial, trigonometric, and inverse functions to model real-world phenomena;
- representing and solving problems using linear programming and difference equations;
- solving systems of linear equations using matrices and vectors;
- describing the concept of continuity of a function;
- performing operations on and between functions; and
- making the connections between trigonometric functions and polar coordinates, complex numbers, and series.

STANDARD 3:

Students use data collection and analysis, statistics, and probability in problem-solving situations and communicate the reasoning used in solving these problems.

In order to meet this standard, a student will

- solve problems by systematically collecting, organizing, describing, and analyzing data using surveys, tables, charts, and graphs;
- make valid inferences, decisions, and arguments based on data analysis; and
- use counting techniques, experimental probability, or theoretical probability, as appropriate, to represent and solve problems involving uncertainty.

RATIONALE

Statistics are used to understand how information is processed and translated into usable knowledge. Through the study of statistics, students learn to collect, organize, and summarize information. Students also need to know how to interpret data and make decisions based on their interpretations. Probability is part of this standard because statistical data are often used to predict the likelihood of future events and outcomes. Students learn probability — the study of chance — so that numerical data can be used to predict future events as well as record the past. A command of statistics and probability is important in adult life.

GRADES K-4

In grades K-4, what students know and are able to do includes

- constructing, reading, and interpreting displays of data including tables, charts, pictographs, and bar graphs;
- interpreting data using the concepts of largest, smallest, most often, and middle;
- generating, analyzing, and making predictions based on data obtained from surveys and chance devices; and
- solving problems using various strategies for making combinations* *(for example, determining the number of different outfits that can be made using two blouses and three skirts).*

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

- reading and constructing displays of data using appropriate techniques *(for example, line graphs, circle graphs, scatter plots*, box plots*, stem-and-leaf plots*)* and appropriate technology;
- displaying and using measures of central tendency*, such as mean, median, and mode, and measures of variability*, such as range and quartiles;
- evaluating arguments that are based on statistical claims;
- formulating hypotheses, drawing conclusions, and making convincing arguments based on data analysis;
- determining probabilities through experiments or simulations;
- making predictions and comparing results using both experimental and theoretical probability drawn from real-world problems; and
- using counting strategies to determine all the possible outcomes from an experiment *(for example, the number of ways students can line up to have their picture taken).*

GRADES 9-12

As students in grades 9-12 extend their knowledge, what they know and are able to do includes

- designing and conducting a statistical experiment to study a problem, and interpreting and communicating the results using the appropriate technology *(for example, graphing calculators, computer software);*
- analyzing statistical claims for erroneous conclusions or distortions;
- fitting curves to scatter plots, using informal methods or appropriate technology, to determine the strength of the relationship between two data sets and to make predictions;
- drawing conclusions about distributions of data based on analysis of statistical summaries *(for example, the combination of mean and standard deviation, and differences between the mean and median);*
- using experimental and theoretical probability to represent and solve problems involving uncertainty *(for example, the chance of playing professional sports if a student is a successful high school athlete)*; and
- solving real-world problems with informal use of combinations and permutations* *(for example, determining the number of possible meals at a restaurant featuring a given number of side dishes).*

For students continuing their mathematics education beyond these standards, what they know and are able to do **may** include

- creating and interpreting discrete and continuous probability distributions, and understanding their application to real-world situations *(for example, insurance);*
- testing hypotheses using appropriate statistics;
- exploring the effect of sample size on the results of statistical surveys using experiments and simulations; and
- solving real-world problems with formal use of combinations and permutations.

STANDARD 4:

Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.

In order to meet this standard, a student will

- connect various physical objects with their geometric representation;
- connect mathematical concepts from across the standards with their geometric representations;
- recognize, draw, describe, and analyze geometric shapes in one, two, and three dimensions;
- make, investigate, and test conjectures about geometric ideas; and
- solve problems and model real-world situations using geometric concepts.

RATIONALE

Long before humans computed, they observed that the full moon, the iris of an eye, and circular ripples of water emanating from a cast stone all have the same shape. Recording and analyzing shapes and their properties eventually gave us the branch of mathematics called geometry. The process continues today as mathematicians develop powerful models of our world (for example, non-Euclidean geometry and fractals*). Students who understand the concepts and language of geometry are better prepared to learn number and measurement ideas as well as other advanced mathematical topics. Students' spatial capabilities frequently exceed their numerical skills and tapping these strengths can foster an interest in mathematics and improve number understandings and skills.*

GRADES K-4

In grades K-4, what students know and are able to do includes

- recognizing shapes and their relationships *(for example, symmetry*, congruence*)* using a variety of materials *(for example, pasta, boxes, pattern blocks);*
- identifying, describing, drawing, comparing, classifying, and building physical models of geometric figures;
- relating geometric ideas to measurement and number sense;
- solving problems using geometric relationships and spatial reasoning* *(for example, using rectangular coordinates* to locate objects, constructing models of three-dimensional objects);* and
- recognizing geometry in their world *(for example, in art and in nature).*

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

- constructing two- and three-dimensional models using a variety of materials and tools;
- describing, analyzing, and reasoning informally about the properties *(for example, parallelism, perpendicularity, congruence)* of two- and three-dimensional figures ;
- applying the concepts of ratio, proportion, and similarity* in problem-solving situations;
- solving problems using coordinate geometry*;
- solving problems involving perimeter and area in two dimensions, and involving surface area and volume* in three dimensions; and
- transforming geometric figures using reflections*, translations*, and rotations* to explore congruence.

GRADES 9-12

As students in grades 9-12 extend their knowledge, what they know and are able to do includes

- finding and analyzing relationships among geometric figures using transformations *(for example, reflections, translations, rotations, dilations*)* in coordinate systems*;
- deriving and using methods to measure perimeter, area, and volume of regular and irregular geometric figures;
- making and testing conjectures about geometric shapes and their properties, incorporating technology where appropriate; and
- using trigonometric ratios* in problem-solving situations *(for example, finding the height of a building from a given point, if the distance to the building and the angle of elevation are known).*

For students continuing their mathematics education beyond these standards, what they know and are able to do **may** include

- deducing properties of figures using vectors*;
- applying transformations, coordinates, and vectors in problem-solving situations; and
- describing, analyzing, and extending patterns produced by processes of geometric change *(for example, limits and fractals).*

STANDARD 5:

Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.

In order to meet this standard, a student will

- understand and apply the attributes of length, capacity*, weight, mass, time, temperature, perimeter, area, volume, and angle measurement in problem-solving situations;
- make and use direct and indirect measurements to describe and compare real-world phenomena;
- understand the structure and use of systems of measurement;
- describe and use rates of change *(for example, temperature as it changes throughout the day, or speed as the rate of change of distance over time)* and other derived measures; and
- select appropriate units, including metric and U. S. customary, and tools *(for example, rulers, protractors, compasses, thermometers)* to measure to the degree of accuracy required to solve a given problem.

RATIONALE

Every day, people measure to answer common questions: How long will it take? How high is it? How much will it hold? Using agreed-upon units, such as inches, paper clips, kilograms, heartbeats, paces, or degrees Celsius, we quantify the world in which we live. Measurement is one way to make numbers meaningful to students. Naturally, measurement is closely allied with geometry (for example, through angular, linear, area, and volume measurements), but measurement involves more than using a ruler and a protractor. Measuring diverse quantities involves making connections within mathematics and across the curriculum.

GRADES K-4

In grades K-4, what students know and are able to do includes

- knowing, using, describing, and estimating measures of length, perimeter, capacity, weight, time, and temperature;

- comparing and ordering objects according to measurable attributes *(for example, longest to shortest, lightest to heaviest);*
- demonstrating the process of measuring and explaining the concepts related to units of measurement;
- using the approximate measures of familiar objects *(for example, the width of your finger, the temperature of a room, the weight of a gallon of milk)* to develop a sense of measurement; and
- selecting and using appropriate standard and non-standard units of measurement in problem-solving situations.

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

- estimating, using, and describing measures of distance, perimeter, area, volume, capacity, weight, mass, and angle comparison;
- estimating, making, and using direct and indirect measurements to describe and make comparisons;
- reading and interpreting various scales including those based on number lines, graphs, and maps;
- developing and using formulas and procedures to solve problems involving measurement;
- describing how a change in an object's linear dimensions affects its perimeter, area, and volume; and
- selecting and using appropriate units and tools to measure to the degree of accuracy required in a particular problem-solving situation.

GRADES 9-12

As students in grades 9-12 extend their knowledge, what they know and are able to do includes

- measuring quantities indirectly using techniques of algebra, geometry, or trigonometry*;
- selecting and using appropriate techniques and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements; and
- determining the degree of accuracy of a measurement *(for example, by understanding and using significant digits).*

For students continuing their mathematics education beyond these standards, what they know and are able to do **may** include

- demonstrating the meanings of area under a curve and length of an arc.

STANDARD 6:

Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem-solving situations and communicate the reasoning used in solving these problems.

In order to meet this standard, a student will

- model, explain, and use the four basic operations — addition, subtraction, multiplication, and division — in problem-solving situations;
- develop, use, and analyze algorithms*; and
- select and apply appropriate computational techniques to solve a variety of problems and determine whether the results are reasonable.

RATIONALE

Computation is an indispensable part of mathematics and our daily lives. We use it to balance our checkbooks, figure our taxes, and make business decisions. The basic facts of addition, subtraction, multiplication, and division are similarly indispensable. Today's students must be able to use a variety of computational tools and techniques including estimation, mental arithmetic, paper-and-pencil, calculators, and computers. Estimation and mental arithmetic serve a practical function in our daily lives, and help students develop meaning for numbers and understanding of number relationships. The use of calculators and computers is not intended to replace proficiency with basic facts. Appropriate uses of calculators and computers include solving real-world problems that may involve tedious or time-consuming computations or exploring number patterns to develop understanding of numbers and number relationships. Proficiency with basic facts is essential for knowing when and how to use each of these tools and techniques.

Computational skill is related to "operation sense". Students with operation sense know when and how to use addition, subtraction, multiplication, and division, and are able to apply them to solve real-world problems. Students build operation sense by modeling their understanding of number operations and their properties, by describing how number operations are related to one another, and by seeing how the use of a particular operation changes the value of the numbers involved.

Computational skill and operation sense go hand in hand with number sense. When children have a well-developed sense of number and operations, they can more easily evaluate the reasonableness of their solutions. The ability to apply computational skills and operation sense will extend students' mathematical power by giving them confidence in their ability to work with numbers and to solve problems in a variety of situations.

GRADES K-4

In grades K-4, what students know and are able to do includes

- demonstrating conceptual meanings for the four basic arithmetic operations of addition, subtraction, multiplication, and division;
- adding and subtracting commonly-used fractions and decimals using physical models *(for example,* $1/3$, $3/4$, *0.5, 0.75);*
- demonstrating understanding of and proficiency with basic addition, subtraction, multiplication, and division facts without the use of a calculator;
- constructing, using, and explaining procedures to compute and estimate with whole numbers; and
- selecting and using appropriate methods for computing with whole numbers in problem-solving situations from among mental arithmetic, estimation, paper-and-pencil, calculator, and computer methods.

GRADES 5-8

As students in grades 5-8 extend their knowledge, what they know and are able to do includes

- using models to explain how ratios, proportions, and percents can be used to solve real-world problems;
- constructing, using, and explaining procedures to compute and estimate with whole numbers, fractions, decimals, and integers;
- developing, applying, and explaining a variety of different estimation strategies in problem-solving situations, and explaining why an estimate may be acceptable in place of an exact answer; and
- selecting and using appropriate methods for computing with commonly used fractions and decimals, percents, and integers in problem-solving situations from among mental arithmetic, estimation, paper-and-pencil, calculator, and computer methods, and determining whether the results are reasonable.

GRADES 9-12

As students in grades 9-12 extend their knowledge, what they know and are able to do includes

- using ratios, proportions, and percents in problem-solving situations;
- selecting and using appropriate methods for computing with real numbers in problem-solving situations from among mental arithmetic, estimation, paper-and-pencil, calculator, and computer methods, and determining whether the results are reasonable; and
- describing the limitations of estimation, and assessing the amount of error resulting from estimation within acceptable limits.

For students continuing their mathematics education beyond these standards, what they know and are able to do **may** include

- analyzing and solving optimization problems*;
- analyzing different algorithms *(for example, sorting)* for efficiency;
- analyzing and using critical path algorithms *(for example, determining in which order to perform a set of tasks in a large project);* and
- investigating problem situations that arise in connection with computer validation and the application of algorithms.

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GLOSSARY

Colorado Model Content Standards for Mathematics

Absolute value — A number's distance from zero on a number line. The absolute value of -6, shown as |-6|, is 6, and the absolute value of 6, shown as |6|, is 6.

Algebra — The branch of mathematics that is the generalization of the ideas of arithmetic.

Algebraic methods — The use of symbols to represent numbers and signs to represent their relationships.

Algorithm — A step-by-step procedure.

Basic facts — Addition facts through 10 (0 + 0, 1 + 0, ... , 10 + 10), subtraction facts which are the inverses of the addition facts (20 - 10, ... , 1 - 0, 0 - 0), multiplication facts (1 x 1, 1 x 2, ... , 10 x 10), and division facts which are the inverses of the multiplication facts (1 ÷ 1, 2 ÷ 1, ... , 100 ÷ 10).

Box plot (also called a box-and-whiskers plot) — A graphic method for showing a summary of data using median, quartiles, and extremes of data. A box plot makes it easy to see where the data are spread out and where they are concentrated. The longer the box, the more the data are spread out.

Capacity — The volume of a container given in units of liquid measure. The standard units of capacity are the liter and the gallon.

Combinations — Subsets chosen from a larger set of objects in which the order of the items doesn't matter (for example, the number of different committees of three that can be chosen from a group of twelve members).

Complex numbers — Numbers that can be written in the form $a + bi$, for example, $-2.7 + 8.9i$, where a and b are real numbers and $i = \sqrt{-1}$.

Congruent or the concept of congruence — Two figures are said to be congruent if they are the same size and shape.

Coordinate geometry — Geometry based on the coordinate system.

Coordinate system (also called rectangular coordinate system) — A method of locating points in the plane or in space by means of numbers. A point in a plane can be located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.

Conjecture — A statement that is to be shown true or false. A conjecture is usually developed by examining several specific situations.

Dilation — A transformation that either enlarges or reduces a geometric figure proportionally.

Exponential function — A function that has an equation of the form $y = a^x$. These functions are used to study population growth or decline, radioactive decay, and compound interest.

Exponent — A number used to tell how many times a number or variable is used as a factor. For example, 5^3 indicates that 5 is a factor 3 times, that is, 5 x 5 x 5. The value of 5^3 is 125.

Fractal — A geometric shape that is self-similar and has fractional dimensions. Natural phenomena such as the formation of snowflakes, clouds, mountain ranges, and landscapes involve patterns. Their pictorial representations are fractals and are usually generated by computers.

Function — A relationship between two sets of numbers (or other mathematical objects). Functions can be used to understand how one quantity varies in relation to another, for example, the relationship between the number of cars and the number of tires.

Geometry — A branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, and two- and three-dimensional figures.

Integers — The set of numbers consisting of the counting numbers (that is, 1, 2, 3, 4, 5, ...), their opposites (that is, negative numbers, -1, -2, -3, ...), and zero.

Irrational numbers — The set of numbers which cannot be represented as fractions. Examples are $\sqrt{2}$, $\sqrt[3]{29}$, e, and π.

Linear function — A function that has a constant rate of change.

Logarithm — Alternate way to express an exponent. For example, $\log_2 8=3$ is equivalent to $2^3=8$.

Matrix (pl. matrices) — A rectangular array of numbers (or letters) arranged in rows and columns.

Measures of central tendency — Numbers which in some sense communicate the "center" or "middle" of a set of data. The mean, median, and mode of statistical data are all measures of central tendency.

Measures of variability — Numbers which describe how spread out a set of data is, for example, range and quartile.

Mental arithmetic — Performing computations in one's head without writing anything down. Mental arithmetic strategies include finding pairs that add up to 10 or 100, doubling, and halving.

Model — To make or construct a physical or mathematical representation.

Number sense — An understanding of number. This would include number meanings, number relationships, number size, and the relative effect of operations on numbers.

Open sentence — a statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown. For example, $3 + x = 5$.

Optimization problems — Real-world problems in which, given a number of constraints, the best solution is determined. For example, finding the best number of nonstop flights from Denver to San Francisco given the cost of fuel, number of passengers, number of crew required, etc.

Patterns — Regularities in situations such as those in nature, events, shapes, designs, and sets of numbers *(for example, spirals on pineapples, geometric designs in quilts, the number sequence 3,6,9,12,...)*.

Permutations — All possible arrangements of a given number of items in which the order of the items makes a difference. For example, the different ways that a set of four books can be placed on a shelf.

Prime number — A counting number that can only be evenly divided by two different numbers, 1 and the number itself. The first ten prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Probability — The likeliness or chance of an event occurring.

Problem solving — Refer to the introduction (page Mathematics - 4).

Problem-solving situations — Contexts in which problems are presented that apply mathematics to practical situations in the real world, or problems that arise from the investigation of mathematical ideas.

Quadratic function — A function that has an equation of the form $y = ax^2+bx+c$, where $a \neq 0$. These functions are used to describe the flight of a ball and the stream of water from a fountain.

Rational numbers — A number that can be expressed in the form a/b, where *a* and *b* are integers and $b \neq 0$, for example, $3/4$, $2/1$, or $11/3$. Every integer is a rational number, since it can be expressed in the form a/b, for example, $5 = 5/1$. Rational numbers may be expressed as fractional or decimal numbers, for example, $3/4$ or .75. Finite decimals, repeating decimals, and mixed numbers all represent rational numbers.

Real numbers — All rational and irrational numbers.

Real-world problems (also called real-world experiences) — Quantitative problems that arise from a wide variety of human experiences which may take into consideration contributions from various cultures (for example, Mayan or American pioneers), problems from abstract mathematics, and applications to various careers (for example, making change or calculating the sale price of an item).

Reflection (also called a flip) — A transformation which produces the mirror image of a geometric figure.

Rotation (also called a turn) — A transformation which turns a figure about a point a given number of degrees.

Scatter plots (also called scatter diagram or scattergram) — A graph of the points representing a collection of data.

Scientific notation — A short-hand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10, for example, $4.53 \times 10^3 = 4350$.

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Similarity — Objects or figures that are the same shape are similar figures. They are not necessarily the same size. If two figures are similar, we say that there is similarity between the figures.

Spatial visualization (also called spatial reasoning) — A type of reasoning in which a person can draw upon one's understanding of relationships in space, the three-dimensional world. For example, spatial reasoning is demonstrated by one's ability to build a three-dimensional model of a building shown in a picture. A person who uses spatial visualization is said to have spatial sense.

Square root — That number which when multiplied by itself produces the given number. For example, 5 is the square root of 25, because $5 \times 5 = 25$.

Statistics — The branch of mathematics which is the study of the methods of collecting and analyzing data. The data are collected on samples from various populations of people, animals, or products. Statistics are used in many fields, such as biology, education, physics, psychology, and sociology.

Stem-and-leaf plot — A frequency distribution made by arranging data. It is one way of visually portraying data that is frequently used in newspapers and magazines because it provides an efficient way of showing information as well as comparing different sets of data.

Symmetry — The correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point. For example, a figure that has line symmetry has two halves which coincide if folded along its line of symmetry.

Transformation — The process of changing one configuration or expression into another in accordance with a rule. Common geometric transformations include translations, rotations, and reflections.

Translation (also called a slide) — A transformation that moves a geometric figure by sliding. Each of the points of the geometric figure moves the same distance in the same direction.

Trigonometric ratios — The ratios of the lengths of pairs of sides in a right triangle. There are three basic trigonometric ratios used in trigonometry: sine (sin), cosine (cos), and tangent (tan).

Trigonometry — A branch of mathematics that combines arithmetic, algebra, and geometry. Trigonometry is used in surveying, navigation, and various sciences such as physics.

Variable — A quantity that may assume any one of a set of values. In the equation $2x + y = 9$, x and y are variables.

Vector — A quantity which has both magnitude and direction. Vectors may be interpreted as physical quantities such as velocity and force.

Volume — The measure of the interior of a three-dimensional figure. A unit for measuring volume is the cubic unit.